

# Minimal Set of Texture Specific Quark Mass Matrices

Samandeep Sharma and Gulsheen Ahuja

**Abstract** Starting with the most general mass matrices, within the context of Standard Model and some of its extensions, incorporating the ideas of weak basis transformations and ‘naturalness’, we find that there exists a particular set of texture specific quark mass matrices which can be considered to be the minimal viable option.

## 1 Introduction

One of the key challenges in the present day high energy physics is to understand the vast spectrum of fermion masses and their relationships with the corresponding mixing angles as well as mass matrices. Despite impressive advances in the measurements of fermion masses and mixing parameters, we are far from having a compelling theory for flavor physics. Even for the case of quarks, where precision measurements are available, the data is understood in terms of phenomenological models having their roots in the ‘bottom up’ approach. In this context, exploring the possibility of finding a minimal set of viable quark mass matrices can perhaps be the first important step for solving the flavor riddle.

The bottom up approach of understanding fermion masses and mixings has evolved in three different directions. Firstly, on the lines of Fritzsch ansatz [1], mass matrices are formulated wherein certain elements of these are assumed to be zero, usually referred to as texture zeros, and the compatibility of the mixing ma-

---

Samandeep Sharma  
Department of Physics, GGDSD College, Sector 32, Chandigarh  
e-mail: sharma.saman87@gmail.com

Gulsheen Ahuja  
Department of Physics, Panjab University, Chandigarh  
e-mail: gulsheenahuja@yahoo.co.in

\*\*Based upon S. Sharma *et al.* Phys.Rev. D91 (2015) 5, 053004.

trix so obtained from these with the low energy data ensures the viability of the formulated mass matrices. However, despite showing considerable promise, in this approach the possibility to arrive at a minimal set of viable quark mass matrices emerges only by carrying out an exhaustive case by case analysis of all possible texture zero mass matrices [2].

Further, within the framework of SM and its extensions, one has the freedom to make unitary transformations, referred to as ‘Weak Basis (WB) transformations’, which change the mass matrices without changing the quark mixing matrix. Using WB transformations, several attempts [3]-[5] have been made wherein the above freedom is exploited to introduce texture zeros in the quark mass matrices. This results in somewhat reducing the number of free parameters of general mass matrices, however, in the absence of any constraints on the elements of the mass matrices, leads to a large number of texture zero matrices which are able to explain the quark mixing data.

In yet another approach, advocated by Peccei and Wang [6], the concept of ‘natural mass matrices’ has been introduced to formulate viable set of mass matrices at the Grand Unified Theories (GUTs) as well as the  $M_Z$  scale. In order to avoid fine tuning, the elements of the mass matrices are constrained in order to reproduce the hierarchical nature of the quark mixing angles. This results in constraining the parameter space available to the elements of the mass matrices, however without yielding a finite set of viable mass matrices at the GUTs as well as the  $M_Z$  scale.

A careful perusal of the above mentioned approaches suggests that none of these leads to a finite set of viable texture specific mass matrices, thus in order to obtain the same perhaps one needs to combine the three. The purpose of the present work is, therefore, to follow the texture zero approach coupled with WB transformations to reduce the number of free parameters of general hermitian mass matrices as well as to impose the condition of ‘naturalness’ for constraining the parameter space available to the elements of these.

## 2 Methodology

To begin with, we consider the following hermitian mass matrices

$$M_q = \begin{pmatrix} E_q & A_q & F_q \\ A_q^* & D_q & B_U \\ F_q^* & B_q^* & C_q \end{pmatrix} \quad (q = U, D), \quad (1)$$

which, without loss of generality, are related to the most general mass matrices [2]. As a next step, one can introduce texture zeros in these matrices using the WB transformations [3], in particular, one can find a matrix  $W$  transforming  $M_U \rightarrow W^\dagger M_U W$  and  $M_D \rightarrow W^\dagger M_D W$ , leading to

$$M_U = \begin{pmatrix} E_U & A_U & 0 \\ A_U^* & D_U & B_U \\ 0 & B_U^* & C_U \end{pmatrix}, \quad M_D = \begin{pmatrix} E_D & A_D & 0 \\ A_D^* & D_D & B_D \\ 0 & B_D^* & C_D \end{pmatrix}. \quad (2)$$

The above matrices, wherein  $A_q = |A_q|e^{i\alpha_q}$  and  $B_q = |B_q|e^{i\beta_q}$  for  $q = U, D$ , can be characterized as texture 2 zero quark mass matrices. Further, in order to incorporate the condition of ‘naturalness’ on these mass matrices, we have considered the following hierarchy for the elements of the matrices [7]

$$(1, i) < (2, j) \lesssim (3, 3); \quad i = 1, 2, 3, \quad j = 2, 3. \quad (3)$$

Therefore, the matrices given in equation (2) can now be considered as most general and their analysis can lead to very broad and interesting consequences.

Before getting into the details of the analysis, we first present some of the essentials pertaining to the construction of the CKM matrix from these mass matrices. Details in this regard can be looked up in [2, 8]. To facilitate diagonalization, for  $q = U, D$ , the mass matrix  $M_q$  may be expressed as  $M_q = Q_q^\dagger M_q^r Q_q$  implying  $M_q^r = Q_q M_q Q_q^\dagger$  where  $M_q^r$  is a symmetric matrix with real eigenvalues and  $Q_q$  is the diagonal phase matrix. The  $M_q^r$  can be diagonalized using the following transformations

$$M_q^{diag} = O_q^T M_q^r O_q = O_q^T Q_q M_q Q_q^\dagger O_q = \text{Diag}(m_1, -m_2, m_3), \quad (4)$$

where the subscripts 1, 2 and 3 refer respectively to  $u, c, t$  for the up sector and  $d, s, b$  for the down sector. Using these diagonalizing transformations, the quark mixing matrix can be obtained from the relation

$$V_{\text{CKM}} = O_U^T Q_U Q_D^\dagger O_D. \quad (5)$$

### 3 Calculations and results

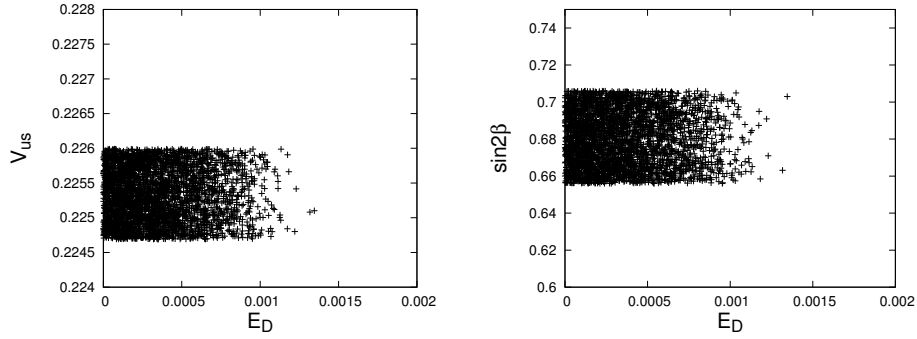
To carry out the numerical work, the parameters  $\phi_1$  and  $\phi_2$ , related to the phases of the mass matrices,  $\phi_1 = \alpha_U - \alpha_D$  and  $\phi_2 = \beta_U - \beta_D$ , have been given full variation from 0 to  $2\pi$ . Apart from  $\phi_1$  and  $\phi_2$ , the free parameters  $E_U, E_D, D_U$  and  $D_D$  have also been given wide variation in conformity with the condition of naturalness. The quark masses and the mass ratios at the  $M_Z$  scale [9] have been used as inputs whereas the latest values [10] of CKM parameters, pertaining to three mixing angles and one CP violating phase, have been used as constraints in our analysis.

Coming to the outcome of our analysis, using the relation between mass matrices and mixing matrix, given in equation (5), the resultant CKM matrix comes out to be

$$V_{\text{CKM}} = \begin{pmatrix} 0.9739 - 0.9745 & 0.2246 - 0.2259 & 0.00337 - 0.00365 \\ 0.2224 - 0.2259 & 0.9730 - 0.9990 & 0.0408 - 0.0422 \\ 0.0076 - 0.0101 & 0.0408 - 0.0422 & 0.9990 - 0.9999 \end{pmatrix}, \quad (6)$$

being fully compatible with the one given by PDG [10]. Also, the CP violating Jarlskog's rephasing invariant parameter  $J$  comes out to be  $(2.494 - 3.365) \times 10^{-5}$  which again is compatible with its latest experimental range,  $(3.06^{+0.21}_{-0.20}) \times 10^{-5}$ .

The viability of these general texture 2 zero mass matrices  $M_U$  and  $M_D$  is beyond doubt, however examining the parameter space available to various elements of these matrices, one finds that the (1,1) element of both  $M_U$  and  $M_D$  is not only quite small in comparison with the other non zero elements but is also essentially redundant. This fact can clearly be understood from a careful look at the Figure (1), wherein the parameter  $E_D$  has been plotted against  $|V_{us}|$  and the CP asymmetry parameter  $\sin 2\beta$ . Similar conclusions can be drawn from  $E_D$  versus the other CKM matrix elements plots. In the up sector, similar plots pertaining to (1,1) element  $E_U$  of the matrix  $M_U$  reveal that again this parameter is also quite small and essentially redundant.



**Fig. 1** Plots showing the dependence of  $V_{us}$  and  $\sin 2\beta$  on the parameter  $E_D$ .

### 3.1 Texture four zero mass matrices

Keeping in mind the above discussion, ignoring the elements  $E_U$  and  $E_D$  of the mass matrices, one gets  $M_U$  and  $M_D$  as

$$M_U = \begin{pmatrix} 0 & A_U & 0 \\ A_U^* & D_U & B_U \\ 0 & B_U^* & C_U \end{pmatrix}, \quad M_D = \begin{pmatrix} 0 & A_D & 0 \\ A_D^* & D_D & B_D \\ 0 & B_D^* & C_D \end{pmatrix}, \quad (7)$$

indicating a transition from texture 2 zero mass matrices to texture 4 zero mass matrices. Carrying out a similar analysis for these matrices, the corresponding CKM matrix comes out to be

$$V_{\text{CKM}} = \begin{pmatrix} 0.9741 - 0.9744 & 0.2246 - 0.2259 & 0.00337 - 0.00365 \\ 0.2245 - 0.2258 & 0.9732 - 0.9736 & 0.0407 - 0.0422 \\ 0.0071 - 0.0100 & 0.0396 - 0.0417 & 0.9990 - 0.9992 \end{pmatrix}. \quad (8)$$

This matrix is not only in complete agreement with the latest quark mixing matrix given by PDG [10], but also is fully compatible with the CKM matrix given in equation (6). Further, the range of the CP violating Jarlskog's rephasing invariant parameter  $J$  comes out to be  $(2.50 - 3.37) \times 10^{-5}$  which again is compatible with its latest experimental range, justifying our earlier conclusion that the elements  $E_U$  and  $E_D$  are essentially redundant as far as reproducing the CKM parameters are concerned.

Further, it is interesting to note that using the WB transformations, apart from the matrices given in equation (7), one gets several other possible texture 4 zero mass matrices which may or may not be related through permutations. Based on whether the matrices are related through permutations or not, all possible texture 4 zero mass matrices can be classified as shown in Table (1). The matrices which are not related to each other through permutations have been put into different categories.

**Table 1** Table showing various phenomenologically allowed texture 2 zero possibilities, categorized into four distinct categories.

	a	b	c	d	e	f
Category 1	$\begin{pmatrix} 0 & A & 0 \\ A^* & D & B \\ 0 & B^* & C \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & A \\ 0 & C & B \\ A^* & B^* & D \end{pmatrix}$	$\begin{pmatrix} D & A & B \\ A^* & 0 & 0 \\ B^* & 0 & C \end{pmatrix}$	$\begin{pmatrix} C & B & 0 \\ B^* & D & A \\ 0 & A^* & 0 \end{pmatrix}$	$\begin{pmatrix} D & B & A \\ B^* & C & 0 \\ A^* & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} C & 0 & B \\ 0 & 0 & A \\ B^* & A^* & D \end{pmatrix}$
Category 2	$\begin{pmatrix} D & A & 0 \\ A^* & 0 & B \\ 0 & B^* & C \end{pmatrix}$	$\begin{pmatrix} D & 0 & A \\ 0 & C & B \\ A^* & B^* & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & A & B \\ A^* & D & 0 \\ B & 0 & C \end{pmatrix}$	$\begin{pmatrix} C & B & 0 \\ B^* & 0 & A \\ 0 & A^* & D \end{pmatrix}$	$\begin{pmatrix} C & 0 & B \\ 0 & D & A \\ B^* & A^* & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & B & A \\ B^* & C & 0 \\ A^* & 0 & D \end{pmatrix}$
Category 3	$\begin{pmatrix} 0 & A & D \\ A^* & 0 & B \\ D^* & B^* & C \end{pmatrix}$	$\begin{pmatrix} 0 & D & A \\ D^* & C & B \\ A^* & B & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & A & B \\ A^* & 0 & D \\ B^* & D^* & C \end{pmatrix}$	$\begin{pmatrix} 0 & B & C \\ B^* & 0 & A \\ C^* & A^* & D \end{pmatrix}$	$\begin{pmatrix} 0 & C & B \\ C^* & D & A \\ B^* & A^* & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & B & A \\ B^* & 0 & C \\ A^* & C^* & D \end{pmatrix}$
Category 4	$\begin{pmatrix} A & 0 & 0 \\ 0 & D & B \\ 0 & B^* & C \end{pmatrix}$	$\begin{pmatrix} C & 0 & B \\ 0 & A & 0 \\ B^* & 0 & D \end{pmatrix}$	$\begin{pmatrix} C & B & 0 \\ B^* & D & 0 \\ 0 & 0 & A \end{pmatrix}$	$\begin{pmatrix} A & 0 & 0 \\ 0 & C & B \\ 0 & B^* & D \end{pmatrix}$	$\begin{pmatrix} D & 0 & B \\ 0 & A & 0 \\ B^* & 0 & C \end{pmatrix}$	$\begin{pmatrix} C & B & 0 \\ B^* & D & 0 \\ 0 & 0 & A \end{pmatrix}$

Coming to the numerical analysis of the matrices listed in the Table, for the matrices belonging to category 1, considering both  $M_U$  and  $M_D$  as 1a type, corresponding to the ones mentioned in equation (7), we have already shown that these are viable and explain the quark mixing data quite well. The other matrices of this category, related through permutation matrix, also yield similar results. For the matrices belonging to category 4, one finds that interestingly these are not viable as in all these matrices one of the generations gets decoupled from the other two. Further, for categories 2 and 3, again a similar analysis reveals that the matrices of these classes are also not viable as can be understood from the following CKM matrices obtained for

categories 2 and 3 respectively, e.g.,

$$V_{CKM} = \begin{pmatrix} 0.9740 - 0.9744 & 0.2247 - 0.2260 & 0.0024 - 0.0099 \\ 0.2205 - 0.2256 & 0.9509 - 0.9727 & 0.0596 - 0.2172 \\ 0.0140 - 0.0445 & 0.0584 - 0.2127 & 0.9905 - 1.0000 \end{pmatrix}, \quad (9)$$

$$V_{CKM} = \begin{pmatrix} 0.9736 - 0.9744 & 0.2247 - 0.2260 & 0.0098 - 0.0331 \\ 0.2226 - 0.2278 & 0.9549 - 0.9719 & 0.0659 - 0.1937 \\ 0.00007 - 0.0340 & 0.0694 - 0.1928 & 0.9810 - 0.9976 \end{pmatrix}, \quad (10)$$

these matrices showing no compatibility with the latest quark mixing data. Therefore, the matrices pertaining to categories 2 and 3 can be considered to be non viable.

## 4 Summary and Conclusions

To summarize, starting with the most general mass matrices, using the concept of weak basis transformations, one first obtains texture 2 zero quark mass matrices. Analysis of these matrices, carried out by incorporating the naturalness condition, reveals that certain elements are essentially redundant, therefore can be discarded, reducing the matrices to texture 4 zero type. Calculations pertaining to all the texture 4 zero mass matrices, related through WB transformations, show that the mass matrices corresponding to the Fritzsch like texture four zero structure and its permutations can be considered to form a minimal set of fermion mass matrices in the quark sector. This minimal set of texture structures for quarks could be the first step towards unified textures for all fermions.

**Acknowledgements** S.S. would like to acknowledge Principal, GGSDS College, Sector 32, Chandigarh and the Chairperson, Department of Physics, P.U., for providing facilities to work.

## References

1. H. Fritzsch, Phys. Lett. B **70**, 436 (1977); **73**, 317 (1978).
2. M. Gupta and G. Ahuja, Int. J. Mod. Phys. A **27**, 1230033 (2012); *ibid* **26**, 2973, (2011).
3. H. Fritzsch and Z. Z. Xing, Phys. Lett. B **413**, 396 (1997); Nucl. Phys. B **556**, 49 (1999).
4. G. C. Branco, D. Emmanuel-Costa and R. Gonzalez Felipe, Phys. Lett. B **477**, 147 (2000); G. C. Branco, D. Emmanuel-Costa, R. Gonzalez Felipe and H. Serodio, Phys. Lett. B **670**, 340 (2009).
5. D. Emmanuel-Costa and C. Simoes, Phys. Rev. D **79**, 073006 (2009).
6. R. D. Peccei and K. Wang, Phys. Rev. D **53**, 2712 (1996).
7. S. Sharma, P. Fakay, G. Ahuja and M. Gupta, Int. J. Mod. Phys. A **29**, 1444005, (2014).
8. S. Sharma, P. Fakay, G. Ahuja and M. Gupta, Phys. Rev. D **91**, 053004 (2015).
9. Z. Z. Xing, H. Zhang and S. Zhou, Phys. Rev. D **86**, 013013 (2012).
10. K. A. Olive *et al.*, Particle Data Group, Chin. Phys. C **38**, 090001 (2014).